Numerical Study on Thermally Induced Birefringence in Yb:YAG Laser Rod: Power Output Optimisation

Dominika Jochcová[1], **Ondřej Slezák**[2]

[1]Department of Physical Electronics, Faculty of Nuclear Sciences and Physical Engineering, Czech Technical University in Prague
[2]Division of High Power Lasers, HiLASE Centre, Institute of Physics, Czech Academy of Sciences
jochcova.dominika@gmail.com

Abstract

A traditional approach to study the birefringence induced under laser pumping is presented. Numerical study of thermally induced birefringence was performed based on material properties of Yb:YAG rod laser crystal in crystallographic orientation [001]. New method introduced here is devoted to optimisation of relative output intensity from the system containing a laser rod placed between two general polarizers.

Keywords: Thermally-induced birefringence; Thermo-optics; Elasto-optics; Laser rod.

Introduction

Generation of the heat in laser crystals is an inevitable process accompanying the generation of laser radiation. The origin of the heat is in non-radiative transitions which occur in a presence of an energy gap between pump band and fluorescent band. Generation of the heat also results from concentration quenching and up-conversion [1, 2]. Spatially distributed heat generation in laser crystal leads to formation of thermal gradients which are non-uniformly altering the index of refraction. Furthermore, heat deposition causes elastic deformation of the material which induces changes in mechanical stresses and the material becomes birefringent due to the photoelastic effect [3, 4, 5]. As a consequence, the material possesses sensitivity to the polarization state of the light passing through [6, 7]. Among its cross-section, the material inhomogeneously change the polarization state of light which leads to decrease of output intensity [8, 9].

In the first part we summarize traditional approach to birefringence estimation in laser rod. The second part is devoted to the method which allows for optimisation of the output intensity.

Theory

Thermally induced birefringence in laser rod

The essentials of photo-elasticity in a geometry of a long cylinder with radial symmetry will be summarized within this section. Let us consider an optical medium of top-hat pumped Yb:YAG rod crystal described by impermeability tensor B_{ij}, which is an inverse of the dielectric tensor ϵ_{ij}. Caused by the phenomena mentioned above, the crystal is heat-loaded, therefore thermally induced stress is generated and impermeability tensor can be described by an isotropic and stress-dependent anisotropic contributions:

$$B_{ij} = B_{0,ij} + \Delta B_{ij}. \tag{1}$$

The isotropic contribution is given by thermally dependant index of refraction with the precision to the first order:

$$B_{0,ij} = 1/\big(n_0 + \frac{dn}{dT}\Delta T\big)^2 \delta_{ij}, \tag{2}$$

where δ_{ij} is the Kroenecker delta and $\Delta T = T(\vec{r}) - T_0$, where T_0 is the stress-free temperature for the crystal. The piezo-optic tensor Π_{ijkl} allows for direct calculation of ΔB_{ij} from knowledge of the stress tensor components σ_{ij}, however Π_{ijkl} is not reliably available for all optical materials, therefore elastic stiffness tensor C_{ijkl} and photoelastic tensor P_{ijkl}, usually easier for measurement, need to be employed:

$$\Delta B_{ij} = \Pi_{ijkl}\sigma_{kl} = P_{ijmn} C^{-1}_{nmkl} \sigma_{kl}. \tag{3}$$

For a long cylinder ($L_0 >> r$) under plain-strain approximation, a cubic crystal can be treated similarly as isotropic medium and analytical solution for thermally-induced stress tensor can be found in literature [10]. For Yb:YAG cubic $m3m$ point group symmetry crystal, the above mentioned tensors can be expressed in Voigt notation in crystallographic orientation [001]:

$$[c] = \begin{bmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{bmatrix}, [p] = \begin{bmatrix} p_{11} & p_{12} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{11} & p_{12} & 0 & 0 & 0 \\ p_{12} & p_{12} & p_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} \end{bmatrix}. \tag{4}$$

When Voigt representation of a four-rank tensor components is desired in a different crystallographic orientation, tensor transform is required and must be performed exceptionally carefully [2]. Next step of the calculation is to restrict field of observation to a plane transverse to the beam propagation direction. This is usually done by reduction of impermeability tensor, see eq. (5).

$$B_{ij} \to B^{\perp} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}. \tag{5}$$

The polarization phase difference $\delta(x, y)$ can be obtained by calculating $n_{1,2}(x, y)$ *i.e.* inverse of the square roots of eigenvalues of submatrix $B^{\perp}$:

$$\delta(x, y) = \frac{2\pi}{\lambda} L(x, y)\big(n_2(x, y) - n_1(x, y)\big), \tag{6}$$

where $L(x, y)$ is thermally dependant rod length, $L(x, y) = L_0(1 + \frac{1}{L_0} dL/dT\Delta T)$ where dL/dT is thermal elongation and λ is laser wavelength. The angle of rotation θ of the principal axis with respect to the laboratory coordinate system is given by:

$$\theta(x, y) = \arccos\big(|\vec{v_1}(x, y)| \cdot |\vec{x_0}|\big), \tag{7}$$

where $\vec{v_1}$ is normalized first or second eigenvector of $B^{\perp}$. Both $\delta(x, y)$ and $\theta(x, y)$ are substituted to the Jones matrix of generally oriented linear phase retarder [11]:

$$\mathbb{M}(x,y) = \begin{bmatrix} \cos\frac{\delta(x,y)}{2} + i\sin\frac{\delta(x,y)}{2}\cos 2\theta(x,y) & i\sin\frac{\delta(x,y)}{2}\sin 2\theta(x,y) \\ i\sin\frac{\delta(x,y)}{2}\sin 2\theta(x,y) & \cos\frac{\delta(x,y)}{2} - i\sin\frac{\delta(x,y)}{2}\cos 2\theta(x,y) \end{bmatrix}. \tag{8}$$

In a polarimetric experiment based on polarizer-analyzer configuration, an optical medium is placed between two crossed linear polarizers. Relative intensity behind this system can be calculated:

$$I_r(x,y) = \sin^2\big(\delta(x,y)/2\big)\sin^2\big(2\theta(x,y) - 2\omega\big), \tag{9}$$

where ω is the angle between laboratory axis and eigen polarization state of the first polarizer.

Optimisation of the power output

The heat-loaded optical medium possesses transversely dependant anisotropy. As a consequence the output polarisation state varies within the medium cross section. However, only one polarisation state can propagate in complex laser system due to sensitivity of optical components such as prisms to polarisation state. Thermally loaded Yb:YAG rod can be described as transversely dependant linear phase retarder, see eq. (8), therefore output polarization state is a function of coordinates (x, y) even if the rod is irradiated with one polarization state. Solution we introduce here is devoted to choice of such output polarizer which maximizes *resp.* minimizes relative output intensity. Let us consider setup based on two ideal elliptical polarizers with an optical medium in between. Polarizer is a projector of a Jones vector *i. e.* $\mathbb{P} = |v\rangle\langle v|$ where $|v\rangle$ is defined as follows:

$$|v(\alpha,\phi)\rangle = \begin{bmatrix} \cos\alpha e^{i\phi} \\ \sin\alpha \end{bmatrix}. \tag{10}$$

In order to take into account every possible polarization state the range of α and ϕ are $[0, \pi/2]$ and $[-\pi, \pi]$ respectively. For further investigation the only relevant characteristic of $|v\rangle$ is that $|v\rangle$ is normalised to unity with respect to p^2 norm $\langle v|v\rangle = 1$. When light passes the system, the distribution of polarization state behind the optical medium is transversely dependent. The figure of merit is total output power *resp.* intensity, therefore the transversal distribution of the polarization state can be ignored:

$$\mathbb{M}(x_j, y_k)|v_{in}(\alpha,\phi)\rangle = |v_{out}(\alpha,\phi,x_j,y_k)\rangle \rightarrow |v_i\rangle. \tag{11}$$

In this discretized problem, the set of states $|v_i\rangle$ is projected into one final polarization state defined by $\mathbb{P}_{out} = |v\rangle\langle v|$. The total output intensity is given by the sum of the set contributions:

$$I_{tot} = \sum_{i=1}^{N} |\mathbb{P}_{out}|v_i\rangle|^2 = \sum_{i=1}^{N} |\,|v\rangle\langle v|v_i\rangle|^2 = \sum_{i=1}^{N} \langle v_i|v\rangle\langle v|v\rangle\langle v|v_i\rangle = \tag{12}$$

$$= \sum_{i=1}^{N} \langle v|v_i\rangle\langle v_i|v\rangle = \langle v|\left(\sum_{i=1}^{N} |v_i\rangle\langle v_i|\right)|v\rangle = \langle v|\mathbb{D}|v\rangle. \tag{13}$$

The linearity of the scalar product and knowledge of the norm of $|v\rangle$ were used within this derivation. The task now is: which $|v\rangle$ should be chosen in order to optimize I_{tot}. $\mathbb{D}$ has two eigen vectors $|u\rangle$, $|w\rangle$ with corresponding eigen values λ and ν, respectively. Consider $\lambda \geq \nu$, then the maximal *resp.* minimal I_{tot} is achieved for $|v\rangle := |u\rangle$ *resp.* $|v\rangle := |w\rangle$. Values λ/N *resp.* ν/N are equal to maximal *resp.* minimal relative intensity which can be acquired.

Results and Discussion

We present here a numerical example of thermally induced birefringence in Yb:YAG laser rod with radius of 5 mm and $L_0 = 5$ cm. The radius of circular top-hat cw pump light is 5 mm, laser wavelength is $\lambda = 1\ \mu$m. In ambient room temperature index of refraction is $n_0 = 1.83$, thermal conductivity is considered constant $k_0 = 11.2$ W$\cdot$m^{-1}K^{-1}, heat transfer coefficient for the rod surface is $h = 20000$ W$\cdot$m^{-2}K^{-1}, thermal elongation $dL/dT = 6.14 \cdot 10^{-6}$ K^{-1}, Young modulus $E = 2.8 \cdot 10^{18}$ Pa and Poisson ratio $\nu = 0.3$ [12]. Values of elastic stiffness and photoelastic tensor are $c_{11} = 3.3 \cdot 10^{11}$ Pa, $c_{12} = 1.65 \cdot 10^{11}$ Pa, $c_{44} = 1.31 \cdot 10^{11}$ Pa, $p_{11} = -0.029$, $p_{12} = 0.0091$ and $p_{44} = -0.0615$ [13]. Figure 1 depicts two visual representation of eq. (9) for a different power absorbed in the rod and a different angle ω.

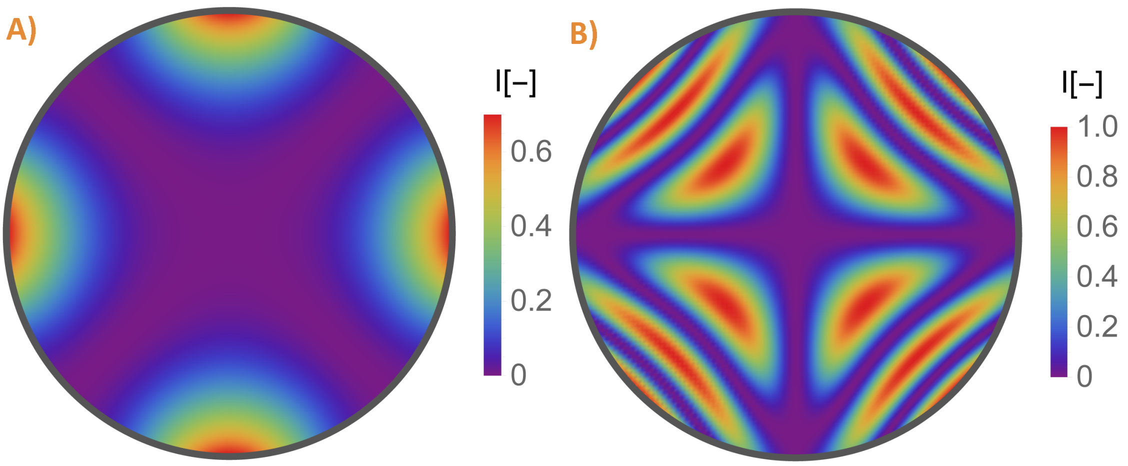

Figure 1: left: $P = 5$ W, $\omega = \pi/4$; right: $P = 20$ W, $\omega = 0$.

Determination of $\delta(x, y)$ and $\theta(x, y)$ allows for direct computation of $\mathbb{M}(x, y)$ via eq. (8). Let us consider linearly polarized input light. In order to prepare linearly polarised states set $\phi = 0$ and range of $\alpha \in [0, \pi/2]$ in eq. (10). Figure 2 C) reveals values of maximal and minimal total intensity $I_{max/min}$ *i.e.* λ/N and ν/N as a function of input polarization state, determined by α. Maximum of total intensity $I_{max} = 0.89$ is achieved for input polarization state corresponding to $\alpha = \pi/4$ and output polarizer $\mathbb{P}_{out} = |v(\pi/4, 0)\rangle \langle v(\pi/4, 0)|$ (here $|u\rangle$ eigen vector of $\mathbb{D}(\alpha = \pi/4)$ is equal to $|v(\pi/4, 0)\rangle$). Figure 2 D) depicts distribution of I_r among the rod cross section for this extremal case. One can point out that Figure 1 A) and Figure 2 D) are complementary. It is not a coincidence - in the first case, output intensity is estimated for laser rod placed between two crossed linear polarizers with $\omega = \pi/4$ and in the second case, input polarization state correspond to the output polarizer eigen mode and $\alpha = \pi/4$, ergo these configurations lead to maximal *resp.* minimal intensity losses of the system.

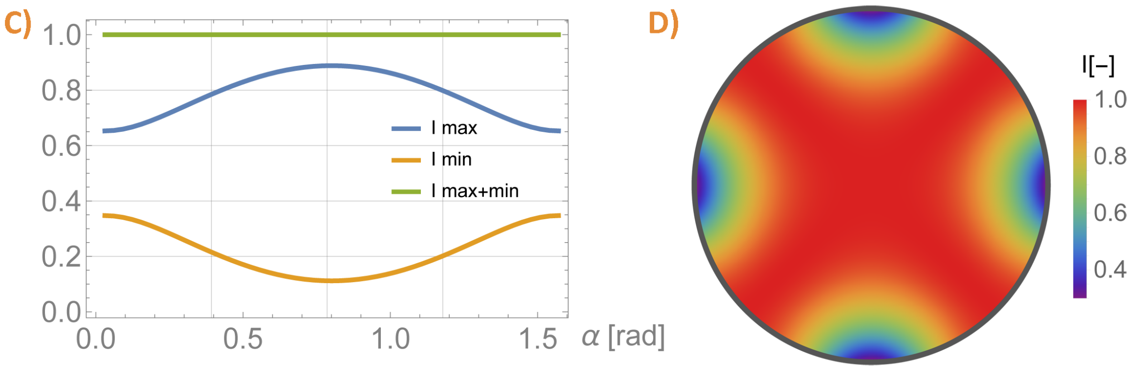

Figure 2: left: eigen values of $\mathbb{D}(\alpha)$; right: maximal intensity profile for $P = 5$ W.

Conclusions

Thermally induced birefringence is an inevitable inconvenience accompanying high power laser systems. A known method was employed in order to determine birefringence in top-hat pumped Yb:YAG laser rod. Subsequently, we introduced a method for optimisation of intensity output from the system based on laser rod placed between two general polarizers. The presented scheme has the potential to be applied as a characterization method for output power optimisation.

Appendix

We present here a proof of the statement that eigen vectors $|u\rangle$ *resp.* $|w\rangle$ maximize *resp.* minimize eq. (13). Considering arbitrary normalized $|v\rangle$, we will show that $\nu \leq I_{tot} \leq \lambda$. Matrix $\mathbb{D}$ is a sum of self-adjoint matrices, therefore $\mathbb{D}$ is self-adjoint which implies that $|u\rangle$ and $|w\rangle$ are orthogonal. $\forall |v\rangle$ we can find $\alpha, \beta \in \mathbb{C}$ such that:

$$|v\rangle = \alpha |u_0\rangle + \beta |w_0\rangle , \tag{14}$$

where $|u_0\rangle$ and $|w_0\rangle$ are normalized eigen vectors of $\mathbb{D}$. Since $|v\rangle$ is normalized, α, β must satisfy $|\alpha|^2 + |\beta|^2 = 1$. Let us investigate $\langle v|\mathbb{D}|v\rangle$ further:

$$\langle v|\mathbb{D}|v\rangle = \alpha^* \alpha \langle u_0|u_0\rangle + \nu \alpha^* \beta \langle u_0|w_0\rangle + \lambda \beta^* \alpha \langle w_0|u_0\rangle + \nu \beta^* \beta \langle w_0|w_0\rangle . \tag{15}$$

First, let us show that expression (15) is greater or equal to ν:

$$\begin{aligned}
\nu &\leq |\alpha|^2 \lambda + |\beta|^2 \nu \\
0 &\leq |\alpha|^2 \lambda + (|\beta|^2 - 1)\nu \\
0 &\leq (1 - |\beta|^2)\lambda - (1 - |\beta|^2)\nu \\
0 &\leq |\alpha|^2 (\lambda - \nu) \qquad \square
\end{aligned}$$

And similarly we can investigate the second inequality:

$$
\begin{aligned}
\lambda &\geq |\alpha|^2\lambda + |\beta|^2\nu \\
0 &\geq (|\alpha|^2 - 1)\lambda + |\beta|^2\nu \\
0 &\geq (\nu - \lambda)|\beta|^2 \qquad \square
\end{aligned}
$$

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